PION COUPLING TO CONSTITUENT QUARKS VERSUS COUPLING TO NUCLEON

Talk presented at the International Workshop
FLAVOR STRUCTURE OF THE NUCLEON SEA
Trento, July 1-5, 2013

MITJA ROSINA

Faculty of Mathematics and Physics, University of Ljubljana,
and Jožef Stefan Institute, Ljubljana
ABSTRACT

The importance of the pion cloud in the nucleon has been demonstrated for several processes and nucleon observables. The most direct observations come from deep inelastic scattering (for example from the forward neutron production in the ep collisions at 300 GeV measured by the H1 and ZEUS Collaborations at DESY). They should be supplemented by forward proton production and we call for more substantial experiments and analyses.

References


OUTLINE

1. INTRODUCTION
2. THE MODEL – PION IN CONSTITUENT QUARKS
3. DEFINITIONS
4. NUCLEON OBSERVABLES
5. ALTERNATIVE PARAMETRIZATION
6. PROTON HAS A NEUTRON+PION COMPONENT
7. EXPERIMENTAL TEST OF PION FLUCTUATION
8. LEADING PROTON PRODUCTION: $p \rightarrow p \pi^0$
9. LEADING PROTON PRODUCTION: $p \rightarrow p \pi^+\pi^-$
10. CONCLUSION
1. INTRODUCTION

Is it a better picture in which each constituent quark contains its own pion cloud, or that with pions coupled directly to a bare nucleon?

Since the amplitudes of pions from different quarks add up (interfere) both pictures seem equivalent. If one takes, however, only the leading multi-pion configuration the descriptions differ, like in comparing jj and LS coupling in nuclear physics.

We show in which cases the ”private pion” description gives a better agreement of different observables with experiment and in which cases both descriptions yield the same values.
Figure 1: Two different coupling schemes

\[
\begin{align*}
\text{u} & + \text{pi} & = & \text{U} \\
\text{d} & + \text{pi} & = & \text{D} \\
\text{p} & + \text{pi} & = & \text{P}
\end{align*}
\]
2. THE MODEL – PION IN CONSTITUENT QUARKS

\[ |u\rangle = \sqrt{1 - \frac{3}{2}a} |u\rangle - \sqrt{a} |d\pi^+\rangle + \sqrt{\frac{a}{2}} |u\pi^0\rangle, \]

\[ |d\rangle = \sqrt{1 - \frac{3}{2}a} |d\rangle + \sqrt{a} |u\pi^-\rangle - \sqrt{\frac{a}{2}} |d\pi^0\rangle. \]

versus – PION CLOUD AROUND BARE NUCLEON

\[ |p\rangle = \sqrt{1 - \frac{3}{2}\alpha} |p\rangle - \sqrt{\alpha} |n\pi^+\rangle + \sqrt{\frac{\alpha}{2}} |p\pi^0\rangle, \]

The basis of pure flavour quarks (and bare nucleons) is denoted by red u, d, p and n.
3. DEFINITIONS
The quantities without superscript $p$ or $n$ refer to proton. Isospin symmetry is assumed: $u^n = d$, $d^n = u$ and $s^n = s$.

$$\hat{u}(x) = u(x) + \bar{u}(x).$$

$$u = u^\uparrow + u^\downarrow, \quad d = d^\uparrow + d^\downarrow, \quad s = s^\uparrow + s^\downarrow;$$

$$\Delta u = u^\uparrow - u^\downarrow, \quad \Delta d = d^\uparrow - d^\downarrow, \quad \Delta s = s^\uparrow - s^\downarrow.$$

$$F^p_1 = \frac{1}{2}(\frac{4}{9}\hat{u} + \frac{1}{9}\hat{d} + \frac{1}{9}\hat{s}), \quad F^n_1 = \frac{1}{2}(\frac{4}{9}\hat{d} + \frac{1}{9}\hat{u} + \frac{1}{9}\hat{s}).$$

$$Gottfried = I_G = 2(F^p_1 - F^n_1) = \frac{1}{3}(\hat{u} - \hat{d}) = \frac{1}{3} + \frac{2}{3}(\bar{u} - \bar{d}).$$

$$I_p = g^p_1 = \frac{1}{2}(\frac{4}{9}\Delta \hat{u} + \frac{1}{9}\Delta \hat{d} + \frac{1}{9}\Delta \hat{s}), \quad I_n = g^n_1 = \frac{1}{2}(\frac{4}{9}\Delta \hat{d} + \frac{1}{9}\Delta \hat{u} + \frac{1}{9}\Delta \hat{s}),$$

$$I_d = \frac{1}{2}(I_p + I_n) = \frac{5}{18}(\Delta \hat{u} + \frac{1}{9}\Delta \hat{d}) + \frac{1}{9}\Delta \hat{s}.$$ 

$$\Delta \Sigma = \Delta \hat{u} + \Delta \hat{d} + \Delta \hat{s}.$$ 

$$g_A^{np} = \Delta \hat{u} - \Delta \hat{d}, \quad g_A^{\Sigma n} = \Delta \hat{d} - \Delta \hat{s}.$$
### 4. NUCLEON OBSERVABLES

<table>
<thead>
<tr>
<th>observable</th>
<th>definition</th>
<th>model value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_A = 1.269 \pm 0.003$</td>
<td>$\Delta u - \Delta d$</td>
<td>$\frac{5}{3}(1 - \frac{4}{3}a)$ = input</td>
</tr>
<tr>
<td>$I_G = 0.216 \pm 0.033$</td>
<td>$\frac{1}{3} + \frac{2}{3}(\bar{u} - \bar{d})$</td>
<td>$\frac{1}{3}(1 - 2a)$ = 0.214</td>
</tr>
<tr>
<td>$I_p = 0.120 \pm 0.017$</td>
<td>$\frac{1}{2}(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d)$</td>
<td>$\frac{5}{18}(1 - \frac{5}{3}a)$ = 0.194</td>
</tr>
<tr>
<td>$I_d = 0.043 \pm 0.006$</td>
<td>$\frac{5}{36}(\Delta u + \Delta d)$</td>
<td>$\frac{5}{36}(1 - 2a)$ = 0.089</td>
</tr>
<tr>
<td>$\Delta \Sigma = 0.330 \pm 0.064$</td>
<td>$\Delta u + \Delta d$</td>
<td>$(1 - 2a)$ = 0.642</td>
</tr>
</tbody>
</table>

Table 1: The $\pi^+$ probability $a = 0.179$ is used.
Notes

- Both models, $(q-\pi)^3$ and $(N-\pi)$, give equal results for the listed observables.

- The strange sea is neglected in this model.

- $g_A$ and the Gottfried sum rule $I_G$ do not feel the strange sea and are in excellent agreement with each other. Other observables obviously lack the strange sea contribution and something else.

- In polarized observables antiquarks do not appear since they appear in pions with equal probability with spin up and spin down.

- We use $I_G$ with screening corrections [?]. Uncorrected values are somewhat higher (about 0.227-0.240 from deep inelastic scattering and 0.255 from Drell-Yan).
5. ALTERNATIVE PARAMETRIZATION

Some authors (Eichten+Hinchliffe+Quigg; Garvey+Peng; Pirner+Povh) presented somewhat different relations for the polarized observables. Probably they tacitly included some contribution of the strange sea. The $\pi^+$ probability has to be fitted again. Some observables like $I_G$ come worse, while the observables sensitive to the strange sea come better.

<table>
<thead>
<tr>
<th>observable</th>
<th>definition</th>
<th>model value</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_A$ = 1.269 ± 0.003</td>
<td>$\Delta u - \Delta d$</td>
<td>$\frac{5}{3}(1 - a)$</td>
<td>input</td>
</tr>
<tr>
<td>$I_G$ = 0.216 ± 0.033</td>
<td>$\frac{1}{3} + \frac{2}{3}(\bar{u} - \bar{d})$</td>
<td>$\frac{1}{3}(1 - 2a)$</td>
<td>0.174</td>
</tr>
<tr>
<td>$I_p$ = 0.120 ± 0.017</td>
<td>$\frac{1}{2}(\frac{4}{9} \Delta u + \frac{1}{9} \Delta d)$</td>
<td>$\frac{5}{18}(1 - 2a)$</td>
<td>0.145</td>
</tr>
<tr>
<td>$I_d$ = 0.043 ± 0.006</td>
<td>$\frac{5}{36}(\Delta u + \Delta d)$</td>
<td>$\frac{5}{36}(1 - 3a)$</td>
<td>0.039</td>
</tr>
<tr>
<td>$\Delta \Sigma$ = 0.330 ± 0.064</td>
<td>$\Delta u + \Delta d$</td>
<td>$(1 - 3a)$</td>
<td>0.283</td>
</tr>
</tbody>
</table>

Table 2: The $\pi^+$ probability $a = 0.239$ is used.
6. PROTON CONTAINS A NEUTRON PLUS PION COMPONENT

The final state $\langle n\pi^+|$ has an overlap with a Fock component of the proton. The result of the explicit calculation is

$$|\langle n\pi^+|p\rangle|^2 = |\langle d\pi^+|u\rangle|^2 = (1 - \frac{3}{2}a) a = 0.13.$$ 

Although the proton has two $u$ quarks there is no factor 2 in the amplitude, due to the flavor-spin-color structure of the nucleon. The flavor-spin wavefunction of the proton has a mixed symmetry combined into a symmetric flavor-spin function:

$$|p\rangle = \sqrt{\frac{1}{2}} \begin{pmatrix} 1 \\ 12 \\ 3 \end{pmatrix}_f \times \begin{pmatrix} 1 \\ 12 \\ 3 \end{pmatrix}_s + \sqrt{\frac{1}{2}} \begin{pmatrix} 1 \\ 13 \\ 2 \end{pmatrix}_f \times \begin{pmatrix} 1 \\ 13 \\ 2 \end{pmatrix}_s.$$
**ISOSPIN FORMALISM**

In the act of producing a positive pion, the corresponding u quark loses one unit of charge, it becomes a d quark. This can be described with the operator $\sum_i t_-(i) = T_-$ where $T_- = T_x - i T_y$. We conveniently took the sum over all three quarks since the third quark, d, contributes zero anyway.

$$
< TM - 1 | T_- | TM > = \sqrt{T(T+1) - M(M-1)}
$$

which for proton ($T = 1/2, M = 1/2$) gives in fact the factor 1. It is instructive to compare with $\Delta^+$ ($T = 3/2, M = 1/2$) in the process $ep \rightarrow e\Delta X$ where one gets the factor 2, pointing out that the two u quarks are always symmetric and interfere constructively.

For the squared amplitude, we get the additional factors: $a \ldots$ from the $\pi^+$-dressed component of the u-quark, $(1 - 3/2a) \ldots$ for the naked component of the final d-quark.
7. EXPERIMENTAL TEST OF PION FLUCTUATION

The analysis of the forward neutron production in the $e^+ p \rightarrow e + n + X$ collisions at 300 GeV measured by the H1 and ZEUS Collaborations at DESY suggests

$$\langle n\pi^+|p\rangle^2 = 0.17 \pm 0.01$$

in reasonable agreement with the above theoretical value

$$|\langle n\pi^+|p\rangle|^2 = |\langle d\pi^+|u\rangle|^2 = (1 - \frac{3}{2} a) a = 0.13.$$
8. LEADING PROTON PRODUCTION: \( p \to p \pi^0 \)

Due to the isospin symmetry,

\[
|\langle p\pi^0 | p \rangle|^2 = \frac{1}{2} |\langle n\pi^+ | p \rangle|^2 = \frac{1}{2}(1 - \frac{3}{2}a) a = 0.065.
\]

In the analysis of leading proton spectrum from DIS at HERA, A. Szczurek, N.N. Nikolaeva and J. Speth (1997) deduced the contributions of pion exchange, reggeon exchange and pomeron exchange.

Our \( \pi^0 \) contribution to the leading proton production is, however, too small. This suggests the importance of two-pion contributions.
Figure 2: Analysis of Szczurek, Nikolaeva and Speth (1997)
9. **LEADING PROTON PRODUCTION:** \( p \rightarrow p \pi^+ \pi^- \)

We again use the isospin formalism. In the act of producing a positive pion, the corresponding u quark loses one unit of charge, it becomes a d quark, and vice versa.

\[
\sum_{i \neq j} t_+(i)t_-(j) = T_+T_- - \sum_i t_+(i)t_-(i) = \vec{T}^2 - T_z^2 - \sum(\vec{t}^2 - t_z^2),
\]

For proton, the expectation value is
\[
\left| \langle p \pi^+ \pi^- | p \rangle \right|^2 = (1 - \frac{3}{2}a)^2a^2 = 0.017.
\]

This result is even smaller than the probability for the \( p \rightarrow p \pi^0 \) production. One possibility is to extend the model.
THE EXTENDED MODEL

We add two-pion configurations to constituent quarks

$$|u\rangle = \sqrt{(1 - \frac{3}{2} a - \frac{3}{2} a^2)} |u\rangle - \sqrt{a} |d\pi^+\rangle + \sqrt{\frac{a}{2}} |u\pi^0\rangle + a |u(\pi^+\pi^- - \pi^0\pi^0 / \sqrt{2})\rangle,$$

$$|d\rangle = \sqrt{(1 - \frac{3}{2} a - \frac{3}{2} a^2)} |d\rangle + \sqrt{a} |u\pi^-\rangle - \sqrt{\frac{a}{2}} |d\pi^0\rangle + a |d(\pi^+\pi^- - \pi^0\pi^0 / \sqrt{2})\rangle.$$

In principle, the amplitude of the two-pion configuration should be a model parameter to be fitted. However, since we are interested only in the qualitative effect, we assumed that each addition of a charged pion brings the same factor $\sqrt{a}$ in the amplitude.
Since the added sea is flavor symmetric (isoscalar) it does not change the Gottfried sum rule. Since it is also isotropic (scalar) it does not contribute to polarization observables. Therefore, no refitting of $a$ is needed.

The main point is, that the amplitudes for kicking two pions from different quarks and from the same quark add coherently.

$$A_{i\neq j} = (1 - \frac{3}{2}a - -\frac{3}{2} a^2) a = 0.122,$$

$$A_{ii} = 3 \times (1 - \frac{3}{2}a - -\frac{3}{2} a^2) a = 0.366,,$$

$$A = A_{i\neq j} + A_{ii} = 0.488, \quad A^2 = 0.24.$$

Such calculation of the two-pion contribution is consistent with their result in the reggeon region. This agreement suggests that their reggeon in fact consists of a scalar two-pion state present in our model. The pomeron exchange is not relevant for our study since it is related to gluon interaction.
10. CONCLUSION

The pion fluctuation $p \rightarrow n + \pi^+$ and $p \rightarrow p + \pi^0$ is an effect of the quark-antiquark pairs of the constituent quarks. The agreement between the measured and the calculated values is a strong support of the constituent quark model.

For the value $a = \langle d\pi^+|u \rangle^2 = 0.18$ each quark contains $0.27 \, q\bar{q} \rightarrow \sim 0.8 \, q\bar{q}$ per nucleon. Using this value of $a$ gives $\langle n\pi^+|p \rangle^2 = 0.13$, to be compared with the experimental value $0.17 \pm 0.01$. It follows that in $\approx 0.26$ cases the proton is a neutron+$\pi^+$ or a proton+$\pi^0$. This means that about one quarter of the nucleon’s quark-antiquark pairs show up as pion fluctuations.

All listed observables (asymmetry $I_G$, polarization observables $g_A, I_p, I_n, \Delta \Sigma$) as well as the one-pion content of proton are the same in both models, the private pion cloud, and the communal pion cloud.

However, the two-pion content as seen in the forward proton process will distinguish the extended model with two-pion admixtures in the constituent quark.
CONCLUSION

\[ |u\rangle = \sqrt{1 - \frac{3}{2}a} |u\rangle - \sqrt{a} |d\pi^+\rangle + \frac{a}{2} |u\pi^0\rangle, \quad |p\rangle = \sqrt{1 - \frac{3}{2}a} |p\rangle - \sqrt{a} |n\pi^+\rangle + \frac{a}{2} |p\pi^0\rangle, \]

versus

Observables \( g_A, I_G, I_p, I_n, \Delta \Sigma \) are the same in both schemes.

Also one-pion probabilities \(<n\pi^+|p>\) and \(<p\pi^0|p>\).

In order to discriminate, one must look at the two-pion probabilities which constitute an important contribution to

FORWARD PROTON PRODUCTION
MINI-WORKSHOP BLED 2013:
LOOKING INTO HADRONS
Bled (Slovenia), July 7-14, 2013

LINK: http://www-f1.ijs.si/Bled2013

CONTACT: Mitja.Rosina@ijs.si
ACKNOWLEDGEMENT

I am pleased to acknowledge the inspiration for this study and the fruitful collaboration with BOGDAN POVH (Max Planck Institute for Nuclear Physics at Heidelberg)

THANKS FOR YOUR ATTENTION!

ISHALL APPRECIATE YOUR CRITICISM AND SUGGESTIONS